Other tests of the isopycnal mixing algorithms have been carried out on various models, applying it to the state variables (temperature and salinity) as well. Generally sharper frontal structures and more accurate (i.e., more vigorous) deep and bottom water formation is produced, due to the reduction of the horizontal component of diffusion across sloping isopycnals in these regions.

It was mentioned earlier that gridpoint noise arising from other terms in the conversation equation may not be satisfactorily suppressed by the isopycnally orientated mixing described here. An extreme example of this arises when it is applied to an "apparent temperature" (buoyancy) type of variable, upon which the density is solely, and linearly dependent. Diffusion of this quantity by the algorithm described above is zero, since the variable is constant along isopycnals by definition. This condition may also arise, to differing degrees, in models treating temperature and salinity independently. In these cases, it is necessary to provide a background horizontal mixing to suppress incipient gridpoint noise. However, the coefficient for this mixing can generally be small compared to that specified for isopycnal mixing.

A modular form of the algorithm is available for use in conjunction with the Cox (1984) version of the Bryan model, and further experimentation with it is encouraged.

REFERENCES

Bryan, K., 1969. A numerical method for the study of the circulation of the World Ocean. J. Comput. Phys., 4, 347-376.

Cox, M. D., 1984. A primitive equation, 3-dimensional model of the ocean. GFDL Ocean Group Tech. Report No. 1, 143 pp.

Cox, M. D., 1985. An eddy resolving numerical model of the ventilated thermocline. J. Phys. Oceanogr., 15, 1312-1324.

Redi, M. H., 1982. Oceanic isopycnal mixing by coordinate rotation. J. Phys. Oceanogr., 12, 1154-1158.

EDDY MIXING AND THE POTENTIAL VORTICITY DISTRIBUTION IN THE SUBTROPICAL THERMOCLINE

Claus W. Böning

A numerical experiment with an eddy-resolving, primitive equation (PE) model by Cox (1985) illustrated the effect of eddy mixing on the distributions of conservative properties. Large areas with uniform tracer and potential vorticity (Q) values appeared on isopycnal surfaces in the subtropical thermocline. The model, driven both by wind- and thermohaline forcing, reproduced various features of the observed oceanic fields of Q (Keffer, 1985). Theoretical approaches toward an understanding of the gyre dynamics have exploited the concept of an almost ideal, Q conserving flow (Rhines and Young, 1982, hereafter R & Y; Luyten et al, hereafter L P S). Within these frameworks, Q homogenization is either explained as an effect of weak eddy mixing in a recirculating, source-free flow, or it requires uniform conditions already at the origin of the water mass, i.e., the formation of pycnostads by water convection.

In collaboration with M. Cox, I have examined the diffusive behaviour of the model flow in the interior of the subtropical gyre, adopting a Lagrangian approach. The analysis of particle trajectories provides a direct view of the transport mechanisms and suggests that homogenization of Q in the model results as an effect of "strong" rather than "weak" eddy mixing.

THE NUMERICAL MODEL

The model under consideration is an 18-level, PE ocean applied over a rectangular basin of 60 degrees width, extending from the equator to 65°N. The depth is constant, 4000 m, except for an idealized shelf and coast along the western wall. The horizontal grid of 1/3° meridional and 0.4° zonal resolution allows the representation of the upper mesoscale range of oceanic eddies. (For details, see Cox, 1985). Note that the model, given its idealized geometry and forcing patterns, should note be regarded as an attempt to "simulate" a real ocean basin. However, with the inclusion of the thermohaline component and the possibility of outcropping isopycnal surfaces in an eddy-resolving study, a step was made which may provide a link between the real ocean and analytical theories, i.e., of R & Y and L P S.

Figure 5 shows the time-mean patterns of eddy kinetic energy (K_E) and Q on an isopycnal surface which outcrops at midlatitude. Fluid enters this surface at the outcrop in the east, carrying with it convectively produced, low Q values. In contrast to a non-eddy-resolving case, also shown by Cox, the advective signature in the Q distribution is wiped out by eddy mixing. The large region of rather uniform, low Q, generated in this way, seperates two areas of high Q associated with the upward displacements and packing of isopycnal surfaces in the Gulf Stream front and along the southeastern flank of the subtropical gyre.

PARTICLE SPREADING

From the time series of velocity $\underset{\sim}{v}$ at the model grid points, 3-dim. trajectories are recovered by numerical integration of

$$\frac{\partial \underset{\sim}{x}(t,p)}{\partial t} = \underset{\sim}{u}(\underset{\sim}{x}(t,p),t)$$

where p identifies a particle, $\underset{\sim}{x}$ is the particle position and $\underset{\sim}{u}$ its 3-D velocity, given by a linear interpolation of $\underset{\sim}{v}$ at the surrounding grid points. The numerical time step used to calculate the trajectories was optimized by taking it to depend on the local velocity value and requiring 200 steps for a particle to cross a grid box (~40 km). In the following, only the lateral component of particle trajectories will be considered.

The spreading of particles illustrates the relative importance of advective and diffusive mechanisms in transporting conservative fluid properties along isopycnal surfaces. Under the assumption of stationary statistics, particle displacements during a time period τ,

$$\underset{\sim}{r}(\tau,p) = \underset{\sim}{x}(t+\tau,p) - \underset{\sim}{x}(t,p)$$

are characterized by the displacement of the center-of-gravity of an ensemble, $\langle \underset{\sim}{r}(\tau) \rangle$, and the turbulent dispersion, defined by the displacement covariance,

$$D_{ij}(\tau) = \langle r_i'(\tau) r_j'(\tau) \rangle$$

where $r' = r - \langle r \rangle$ are individual displacements relative to the mean motion.

A strong role of eddy mixing is apparent from the particle behaviour in the subtropical gyre. Figure 6 shows, as an example, positions of particles 0.5 and 1.5 years after release in the eastern, "ventilated" portion of the thermocline. Particles are spread over a large fraction of the gyre before they are carried to the western boundary by the gyre-scale flow. Figure 7 indicates ensemble-mean displacements in the upper thermocline during a 6 month period, together with the principle axis of the displacements are of the same order of magnitude as the displacement by the mean flow.

A measure of the relative importance of mean advection and eddy dispersal on total property transport may be given by the ratio of mean to rms-displacement,

$$B(t) = \frac{<|\underset{\sim}{r}(t)|>}{(D_{11}^{1/2} \cdot D_{22}^{1/2})^{1/2}} \tag{1}$$

Figure 8 shows B after 6 months of spreading time. Groups consisting of 144 particles in each model level, have been released at different times during a 6 year interval in the westward flowing portion of the subtropical gyre. The values represent averages over 10 realizations. The decrease of B in the thermocline reflects the different depth dependencies of the mean and fluctuating component of the flow, with a more barotropic nature of the eddy field. With this vertical structure, the relative influence of eddy mixing has to be strongest at the base of the thermocline.

Note the time dependence of B: in general, it will grow with t and the advective component will become more and more dominant. (In the ideal case of homogeneous turbulence, $D \sim t^{1/2}$, and a constant mean flow, $< r > \sim t$, we would expect $B \sim t^{1/2}$). Analogous information is given by the Peclet number, Pe = UL/K, in the Eulerian framework. Using the diffusivity values K discussed below, Pe shows a similar depth dependence as R in the subtropical thermocline. A "local" Pe, specifying the length scale as L = 300 km, has a near surface maximum of about 3.

EDDY DIFFUSIVITY

The time-rate-of-change of single particle dispersion defines the Lagrangian diffusivity

$$\underset{\sim}{K} = \frac{1}{2} \frac{d}{dt} \underset{\sim}{D}$$

In the general case, K is a function of spreading time and cannot be regarded as a property of the flow field at a given location, i.e., it cannot be interpreted as an "eddy diffusivity". A special situation, which is much easier to interpret, occurs under the condition of statisitical homogeneity and stationarity. In this case, the dispersion of particles is related to the Lagrangian velocity autocorrelation R(t)

$$D_{ij}(t) = (\overline{u_i'^2} \ \overline{u_j'^2})^{1/2} \int_0^t (t-s) \, [R_{ij}(s) + R_{ji}(s)] \, ds \tag{3}$$

(Batchelor, 1949), and, when and if the motion becomes decorrelated, $D(t) \sim t$ or K = constant.

The significance of the homogeneous case lies in the fact that here $\underset{\sim}{K}$ can be interpreted in terms of the eddy diffusivity tensor defined in a Eulerian flux-gradient relation. From the pattern of eddy kinetic energy in an ocean basin it is apparent, however, that in many regions, especially near the western boundary, the assumption of homogeneity of the eddy field is fundamentally wrong (and an eddy diffusivity concept may not make any sense at all).

The situation seems to be more favourable in our main area of interest, the interior of the subtropical gyre. The model basin is wide enough to accommodate a broad eastern region of relatively weak mean flow without substantial shear. The eddy field is charaterised by a band of moderate intensity with weak gradients in the mean flow direction.

A test of (3) in this region is given with

Figure 9 which compares the Lagrangian diffusivity as derived directly from particle dispersion, $K(t) = 1/2\, d/dt\, D(t)$ with the "diffusivity" as obtained from the velocity correlation,

$$K^*(t) = \overline{u'^2} \int_0^t R(s)\, ds.$$

The diffusivities in the thermocline exhibit a marked transition from an initial linear increase to nearly time-independent values after about 30 days. There is a good agreement between the actual and predicted curves up to about 100 days when the zonal diffusivity begins to rise sharply. This break in the random walk behaviour sets a limit for the applicability of the concept of a constant eddy diffusivity in this flow regime. An enhanced dispersion in the direction of the mean flow has to occur as an effect of lateral mean shear which can be expected to become a dominant dispersion mechanism for longer time scales. Particle spreading in the weaker eddy field below the thermocline indicates a more wave-like behaviour. Consistent with negative lobes in the velocity correlation (Figure 11), especially the meridional diffusivity decreases before approaching a random walk type behaviour.

In order to portray the depth dependence of particle dispersion in this region, eddy diffusivities are defined as time mean values of K(t) between days 40 and 100 (Figure 10). The values represent averages over ten ensembles with 144 particles in each level, released within a six year period. The error bars represent the standard deviation of the latter averaging procedure and indicate the variability in time. Similar to the velocity variance, the diffusivity generally decreases, from maximum values in the thermocline to almost depth independent values below the thermocline. Despite the near isotropy of the eddy velocity field, the diffusivity is anisotropic, especially in the deeper levels. The change in particle spreading is illustrated in the velocity autocorrelation functions (Figure 11) which indicate a rather turbulent flow character in the thermocline and a more wave-like behaviour in the deeper levels. This signature indicates a dominance of zonal motions in the low-frequency part of the velocity spectrum. The Lagrangian behaviour is consistent with the zonal bands of weakly depth dependent flow in the model as found by Cox (1987). The change in particle statistics with depth appears as a consequence of the more barotropic nature of the bands as compared to the more baroclinic nature of the eddies. (A recently performed sensitivity experiment with a rough bathymetry indicates, however, that this particular model behaviour is a consequence of the flat bottom with strong intermodal energy exchange and a rather weak depth dependence of K_E.)

SIGNIFICANCE OF THE MODEL RESULTS

Can the model behaviour tell us something about the transport mechanisms in the oceanic thermocline? Given its idealized geometry and forcing patterns, the model should not be regarded as a "simulation" of a real ocean basin. Notwithstanding some major problems, e.g., in reproducing the distribution of eddy kinetic energy (K_E), the model shows, at least, a realistic eddy intensity in the interior, westward flowing portion of the subtropical thermocline. From satellite-tracked surface drifters, Krauss and Käse (1984) derived K_E values below 100 cm^2s^{-2} south of about 40°N in the central/eastern North Atlantic. With an extended data base, Krauss and Böning (1987) report minimum values of about 70 cm^2s^{-2} at 30°N. Current meter records, however sparse in this area, give eddy energies generally below 100 cm^2s^{-2} in the subtropical eastern NA thermocline (Dickson, 1983).

Diffusivity estimates for this region are even more sparse, but they seem to give a consistent picture. From surface drifter dispersion, Krauss and Böning (1987) derived $(K_x, K_y) = (2.5 * 10^7, 2.1 * 10^7\ cm^2s^{-1})$ at 30°N. Modelling observed tracer distributions on isopycnal surfaces with a prescribed climatological mean flow, Thiele et al, (1986) arrived at "best fit" eddy diffusivities of $2.9*10^7$ north and $1.7*10^7$ south of 29°N. Whereas these values are smaller than those of the model (especially K_x), this does not necessarily mean a weaker effect of eddy mixing which may be better characterized by the ratio of advection to diffusion. Thiele et al, (1986) report "local" Pe ~2 (based on L = 300 Km). Taking the same length scale, the transport in the model thermocline is characterized by Pe ~2-4, suggesting a comparable or even weaker diffusive influence.

The numerical model demonstrates that such a "locally strong" eddy mixing can effectively shape distributions of conservative properties even on a gyre scale. The large uniform areas of tracer and Q in the model thermocline appear as consequences of a rapid mixing of subducted fluid with recirculating "pool" waters on its path into the gyre interior. These results suggest that the assumption of an almost inviscid fluid might not be appropriate when considering tracer transport in the ocean.

REFERENCES

Batchelor, G. K., 1949. Diffusion in a field of homogeneous turbulence. I. Eulerian Analysis. Aust. J. Sci. Res., A2, 437-450.

Cox, M., 1985. An eddy resolving numerical model of the ventilated thermocline. J. Phys. Oceanogr., 15, 1312-1324.

Cox, M., 1987. An eddy resolving model of the ventilated thermocline: time dependence. J. Phys. Oceanogr. (to appear).

Dickson, R. R. 1983. Global summaries and intercomparisons: Flow statistics from long-term current meter moorings. In: A. R. Robinson (Ed), Eddies in Marine Science, Springer Verlag.

Keffer, T., 1985. The ventilation of the world's oceans: Maps of the potential vorticity field. J. Phys. Oceanogr., 15, 509-523.

Krauss, W., & Böning, C. W., 1987. Lagrangian properties of eddy fields in the northen North Atlantic as deduced from satellite-tracked buoys. J. Mar. Res. (to appear).

Krauss, W., & Käse, R. H., 1984. Mean circulation and eddy kinetic energy in the eastern North Atlantic. J. Geophys. Res., 89, 3407-3415.

Luyten, J. R., Pedlosky, J., & Stommel, H., 1983. The ventilated thermocline. J. Phys. Oceanogr., 13, 292-309.

Rhines, P. B., & Young, W., 1982. A theory of wind-driven ocean circulation, I. Mid-ocean gyres. J. Mar. Res., 40, (Suppl.) 559-596.

Thiele, G., et al., 1986. Baroclinic flow and transient-tracer fields in the Canary-Cape Verde Basin. J. Phys. Oceanogr., 16, 814-826.